About direct problems of lava flow: Although lava flows do not significantly affect the life of people, its hazard is not negligible as hot lava kills vegetation, destroys infrastructure, and may trigger a flood due to melting of snow/ice. The lava flow hazard can be reduced if the flow patterns are known, and the complexity of the flow with a debris is analyzed to assist in disaster risk mitigation. We develop three-dimensional numerical models of a gravitational flow of multi-phase fluid with rafts (mimicking rigid lava-crust fragments) on a horizontal and topographic surfaces to explore the dynamics and the interaction of lava flows. We have obtained various flow patterns and spatial distribution of rafts depending on conditions at the surface of fluid spreading, obstacles on the way of a fluid flow, raft landing scenarios, and the size of rafts. Furthermore, we analyze two numerical models related to specific lava flows: (i) a model of fluid flow with rafts inside an inclined channel, and (ii) a model of fluid flow from a single vent on an artificial topography, when the fluid density, its viscosity, and the effusion rate vary with time. Although the studied models do not account for lava solidification, crust formation, and its rupture, the results of the modeling may be used for understanding of flows with breccias before a significant lava cooling.

Data assimilation (inverse problems) in lava flow: Once lava moves far from the center of volcanic magma eruption, its solidifies and a crust develops on the top of the lava flow. Hence, its physical properties (temperature and dynamics) in the lava's interior become unknown. We study a model of lava flow to determine its thermal and dynamic characteristics from thermal measurements of the lava at its surface. Mathematically this problem is reduced to solving an inverse boundary problem. Namely, using known conditions at one part of the model boundary we determine the missing condition at the remaining part of the boundary. We develop a numerical approach to the mathematical problem in the case of steady-state flow.

Assuming that the temperature and the heat flow are prescribed at the upper surface of the model domain, we determine the flow characteristics in the entire model domain using an adjoint method. We have performed computations of model examples and showed that in the case of smooth input data the lava temperature and the flow velocity can be reconstructed with a high accuracy. As expected, a noise imposed on the smooth input data results in a less accurate solution, but still acceptable below some noise level. Also we analyse the influence of optimization methods on the solution convergence rate. The proposed method for reconstruction of physical parameters of lava flows can also be applied to other problems in geophysical fluid flows.